11-7 Adding and Subtracting Radical Expressions

Warm Up

Lesson Presentation

Lesson Quiz
Warm Up

Simplify each expression.

1. \(14x + 15y - 12y + x\) \(\rightarrow 15x + 3y\)

2. \(9xy + 2xy - 8xy\) \(\rightarrow 3xy\)

3. \(-3(a + b) + 5\left(2 + \frac{2}{5}b\right)\) \(\rightarrow -3a - b + 10\)
Warm Up Continued

Simplify. All variables represent nonnegative numbers.

4. \( \sqrt{96} \)
   \[ 4\sqrt{6} \]

5. \( \sqrt{x^9y^{10}} \)
   \[ x^4y^5\sqrt{x} \]

6. \( \frac{\sqrt{72a^5}}{81} \)
   \[ \frac{2a^2\sqrt{2a}}{3} \]
Objective

Add and subtract radical expressions.
Vocabulary

like radicals
Square-root expressions with the same radicand are examples of **like radicals**.

<table>
<thead>
<tr>
<th>Like Radicals</th>
<th>$2\sqrt{5}$ and $4\sqrt{5}$</th>
<th>$6\sqrt{x}$ and $-2\sqrt{x}$</th>
<th>$3\sqrt{4t}$ and $\sqrt{4t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unlike Radicals</td>
<td>$2$ and $\sqrt{15}$</td>
<td>$6\sqrt{x}$ and $\sqrt{6x}$</td>
<td>$3\sqrt{2}$ and $2\sqrt{3}$</td>
</tr>
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</table>
Like radicals can be combined by adding or subtracting. You can use the Distributive Property to show how this is done:

\[ 2\sqrt{5} + 4\sqrt{5} = (2 + 4)\sqrt{5} = 6\sqrt{5} \]

\[ 6\sqrt{x} - 2\sqrt{x} = (6 - 2)\sqrt{x} = 4\sqrt{x} \]

Notice that you can combine like radicals by adding or subtracting the numbers multiplied by the radical and keeping the radical the same.
Adding and Subtracting Radical Expressions

Combining like radicals is similar to combining like terms.

\[ 2\sqrt{5} + 4\sqrt{5} = 6\sqrt{5} \]
\[ 2x + 4x = 6x \]

Helpful Hint
Example 1: Adding and Subtracting Square-Root-Root Expressions

Add or subtract.

A. $9\sqrt{3} - 4\sqrt{3}$

$9\sqrt{3} - 4\sqrt{3}$

$5\sqrt{3}$  

The terms are like radicals.

B. $6\sqrt{x} + 8\sqrt{y}$

$6\sqrt{x} + 8\sqrt{y}$

The terms are unlike radicals. Do not combine.
Example 1: Adding and Subtracting Square-Root-Root Expressions

Add or subtract.

C. \( \sqrt{m} - 7\sqrt{m} \)

\( 1\sqrt{m} - 7\sqrt{m} \)

\( \sqrt{m} = 1\sqrt{m} \), the terms are like radicals.

D. \( 2\sqrt{xy} + 2\sqrt{y} + 9\sqrt{xy} \)

\( 2\sqrt{xy} + 2\sqrt{y} + 9\sqrt{xy} \)

Identify like radicals.

\( 11\sqrt{xy} + 2\sqrt{y} \)

Combine like radicals.
Check It Out! Example 1

Add or subtract.

a. $5\sqrt{7} - 6\sqrt{7}$

$5\sqrt{7} - 6\sqrt{7}$

$\sqrt{7}$

The terms are like radicals.

b. $8\sqrt{3} - 5\sqrt{3}$

$8\sqrt{3} - 5\sqrt{3}$

$3\sqrt{3}$

The terms are like radicals.
11-7 Adding and Subtracting Radical Expressions

Check It Out! Example 1

Add or subtract.

c. $4\sqrt{n} + 4\sqrt{n}$

$4\sqrt{n} + 4\sqrt{n}$

$8\sqrt{n}$

The terms are like radicals.

d. $\sqrt{2s} - \sqrt{5s} + 9\sqrt{5s}$

$\sqrt{2s} - 1\sqrt{5s} + 9\sqrt{5s}$

$\sqrt{y} = 1\sqrt{y};$ The terms are like radicals.

$\sqrt{2s} + 8\sqrt{5s}$

Combine like radicals.
Sometimes radicals do not appear to be like until they are simplified. Simplify all radicals in an expression before trying to identify like radicals.
Example 2A: Simplify Before Adding or Subtracting

Simplify each expression.

\[ \sqrt{45} - \sqrt{20} \]

\[ \sqrt{9 \cdot 5} - \sqrt{4 \cdot 5} \]

Factor the radicands using perfect squares.

\[ \sqrt{9} \sqrt{5} - \sqrt{4} \sqrt{5} \]

Product Property of Square Roots.

\[ 3 \sqrt{5} - 2 \sqrt{5} \]

Simplify.

\[ \sqrt{5} \]

Combine like radicals.
Example 2B: Simplify Before Adding or Subtracting

Simplify each expression.

\[ 9\sqrt{75} + 2\sqrt{50} \]

Factor the radicands using perfect squares.

\[ 9\sqrt{3(25)} + 2\sqrt{2(25)} \]

Product Property of Square Roots.

\[ 9\sqrt{3}\sqrt{25} + 2\sqrt{2}\sqrt{25} \]

Simplify.

\[ 9(5)\sqrt{3} + 2(5)\sqrt{2} \]

The terms are unlike radicals. Do not combine.

\[ 45\sqrt{3} + 10\sqrt{2} \]
Example 2C: Simplify Before Adding or Subtracting

Simplify each expression.

\[ \sqrt{75y} - 2\sqrt{27y} + \sqrt{48y} \]

\[ \sqrt{25(3y)} - 2\sqrt{9(3y)} + \sqrt{16(3y)} \]

\[ 5\sqrt{3y} - 2(3)\sqrt{3y} + 4\sqrt{3y} \]

\[ 5\sqrt{3y} - 6\sqrt{3y} + 4\sqrt{3y} \]

\[ 3\sqrt{3y} \]
Remember!
When you write a radicand as a product, make at least one factor a perfect square.
Check It Out! Example 2a

Simplify each expression.

$$\sqrt{54} + \sqrt{24}$$

$$\sqrt{9(6)} + \sqrt{4(6)}$$

Factor the radicands using perfect squares.

$$\sqrt{9}\sqrt{6} + \sqrt{4}\sqrt{6}$$

Product Property of Square Roots.

$$3\sqrt{6} + 2\sqrt{6}$$

Simplify.

$$5\sqrt{6}$$

Combine like radicals.
Check It Out! Example 2b

Simplify each expression.

\[ 4\sqrt{27} - \sqrt{18} \]

\[ 4\sqrt{9(3)} - \sqrt{9(2)} \quad \text{Factor the radicands using perfect squares.} \]

\[ 4\sqrt{9}\sqrt{3} - \sqrt{9}\sqrt{2} \quad \text{Product Property of Square Roots.} \]

\[ 4(3)\sqrt{3} - 3\sqrt{2} \quad \text{Simplify.} \]

\[ 12\sqrt{3} - 3\sqrt{2} \quad \text{The terms are unlike radicals. Do not combine.} \]
Check It Out! Example 2c

Simplify each expression.

\[ \sqrt{12y} + \sqrt{27y} \]

\[ \sqrt{4(3y)} + \sqrt{9(3y)} \quad \text{Factor the radicands using perfect squares.} \]

\[ \sqrt{4} \sqrt{3y} + \sqrt{9} \sqrt{3y} \quad \text{Product Property of Square Roots.} \]

\[ 2\sqrt{3y} + 3\sqrt{3y} \quad \text{Simplify.} \]

\[ 5\sqrt{3y} \quad \text{Combine like radicals.} \]
Example 3: Geometry Application

Find the perimeter of the triangle. Give the answer as a radical expression in simplest form.

\[ 10 + 13\sqrt{5} + 3\sqrt{20} \]

Write an expression for perimeter.

\[ 10 + 13\sqrt{5} + 3\sqrt{4(5)} \]

Factor 20 using a perfect square.

\[ 10 + 13\sqrt{5} + 3\sqrt{4} \sqrt{5} \]

Product Property of Square Roots.

\[ 10 + 13\sqrt{5} + 3(2) \sqrt{5} \]

Simplify.

\[ 10 + 13\sqrt{5} + 6\sqrt{5} \]

Combine like radicals.

\[ 10 + 19\sqrt{5} \]

The perimeter is \((10 + 19\sqrt{5})\) mm.
Check It Out! Example 3

Find the perimeter of a rectangle whose length is \(2\sqrt{b}\) inches and whose width is \(3\sqrt{b}\) inches. Give your answer as a radical expression in simplest form.

\[
2 \left( 3\sqrt{b} + 2\sqrt{b} \right)
\]

\[
= 2(6\sqrt{b} + 4\sqrt{b})
\]

\[
= 10\sqrt{b}
\]

The perimeter is \(10\sqrt{b}\) in.
Lesson Quiz: Part I

Add or subtract.

1. \(18\sqrt{6} + 22\sqrt{6}\)  \(40\sqrt{6}\)

2. \(3\sqrt{x} + 3\sqrt{3x}\)  \(3\sqrt{x} + 3\sqrt{3x}\)

3. \(-4\sqrt{11y} - 14\sqrt{10y} + 2\sqrt{10y} + \sqrt{11y}\)  \(-3\sqrt{11y} - 12\sqrt{10y}\)

Simplify each expression.

4. \(\sqrt{242} - \sqrt{128}\)  \(3\sqrt{2}\)

5. \(\sqrt{108yz} + 3\sqrt{98yz} + 2\sqrt{75yz}\)  \(16\sqrt{3yz} + 21\sqrt{2yz}\)
6. Find the perimeter of the trapezoid. Give the answer as a radical expression in simplest form.

\[(10\sqrt{6} + 2)\]